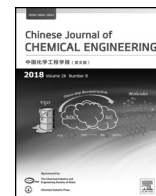




Contents lists available at ScienceDirect

Chinese Journal of Chemical Engineering

journal homepage: www.elsevier.com/locate/CJChE

Article

Prediction model of slurry pH based on mechanism and error compensation for mineral flotation process☆

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ARTICLE INFO

Article history:

Received 30 October 2017

Received in revised form 2 February 2018

Accepted 11 June 2018

Available online 21 June 2018

Keywords:

Froth flotation

PH value prediction

Hydrolysis

Mechanism model

ARMA

Expert rule

ABSTRACT

A suitable pH value of the slurry is a key to efficient mineral flotation. Considering the control delay problem of pH value caused by offline pH measurement, an integrated prediction model for pH value in bauxite froth flotation is proposed, which considers the effect of ore compositions on pH value. Firstly, a regression model is obtained for alkali (Na_2CO_3) consumed by the reaction between ore and alkali. According to the first-order hydrolysis of the remaining alkali, a mechanism-based prediction model is presented for the pH value. Then, considering the complexity of the flotation mechanism, an error prediction model which uses time series of the error of the mechanism model as inputs is presented based on autoregressive moving average (ARMA) method to compensate the mechanism model. Finally, expert rules are established to correct the error compensation direction, which could reflect the dynamic changes during the process accurately and effectively. Simulation results using industrial data show that the presented model meets the needs of the industrial process, which laid the foundation for predictive control of pH regulator.

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1. Introduction

In mineral processing, the pH value of slurry has a direct impact on the compositions of the ions in the slurry, the activity of the flotation reagents and the floatability of the minerals [1]. Only with appropriate pH value, the process could get optimal flotation performance. However, usually, from the location where a pH regulator is added to where it is detected, there exists a time delay that would cause the control delay of the pH value. Moreover, due to the poor environment and the quick deposit of the solid particles, online measurement of pH value is easy to become inaccurate and the equipment is easily jammed and damaged [2]. Then, the dynamic process cannot be monitored in real-time, which exacerbates fluctuations in pH value. In the actual production process, operators detect pH value offline and adjust reagent addition manually to control slurry pH, which leads to heavy workload, bad real-time performance and unstable working conditions. Therefore, predicting the pH value of slurry accurately and real-time has great significance for optimization of flotation process.

In saline mineral flotation process, the slurry pH is not only concerned with the pH regulator and water addition, but also related to

ore compositions. Dissolved ore components may have a great influence on flotation [3]. The relationship between the ore and the pH regulator is complex. In order to find optimal pH range in flotation process, the recovery rate and concentrate grade of useful minerals under different pH values have been studied [4–6]. However, for the bauxite, the research on the effect of ore to pH regulator is very rare, and then the pH regulator addition almost entirely depends on operators' experience.

Soft-sensing and prediction modeling are effective ways to solve the problem that the process variables are difficult to detect on-line. Considering the static models would become more and more inaccurate as time passes, adaptive methods [7–11], or model (parameters) updating strategy [12, 13] and dynamic modeling methods [14–17] are usually used to improve model adaptability. The above methods could improve the model performance to some extent, but it has its applicable conditions [18]. Moreover, there is still much work to do on nonlinear and dynamic modeling. As to the slurry pH, in ref. [19], a soft-sensing model is proposed, which use the froth surface features as auxiliary variables, however, this model cannot predict pH value accurately when the process variables, such as dosage, ore input or the water flow rate, change suddenly. Meanwhile, these models are essentially for soft-sensing, but not suitable for prediction. Flotation is a long process, and there are many external factors that interfere with the interactions of pH regulator and ore, which makes the pH value change in a strong-nonlinear and dynamic behavior. Therefore, further studies on prediction and control are required.

In this paper, to solve the detection delay problem of slurry pH in bauxite froth flotation, an integrated prediction model for slurry pH is

☆ Supported by the National Natural Science Foundation of China (61673401), the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (61621062), the Fundamental Research Funds for the Central Universities of Central South University (2016zzts343).

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established. Firstly a regression model is obtained for alkali (Na_2CO_3) consumed by the reaction between ore and alkali. Secondly, considering the first-order hydrolysis of the remaining alkali, a mechanism-based prediction model for pH value is presented. And then, according to the time series of the pH error which is the difference between the measured value and the predicted value of the mechanistic model, an error compensation model is established based on autoregressive moving average (ARMA). Finally, expert rules are presented to correct the error compensation direction according to the dynamic changes in working conditions. Simulation results using industrial data demonstrate the effectiveness of the proposed method.

2. Influence Factors of pH Value in Flotation Process

In order to obtain a suitable pH value for the slurry in flotation, a pH regulator (Na_2CO_3) is added in the grinding stage. For the purpose of gaining an appropriate particle size and slurry density for flotation, water is added and adjusted in grinding and classification process. To save resources, most of the water added is circulating water whose pH value is around 8.5. More complicated, some minerals in the ore are soluble in water, which causes the slurry to become acidic or alkaline. Some of the resolved minerals react with acidic or alkaline directly, which increases the mineral solubility and consumes some of the pH regulator. Therefore, the pH regulator consumption is related to its own concentration, ore compositions and temperature.

In the industrial process, flotation operators detect the pH value of sampled slurry firstly, and then inform dosing workers to adjust the amount of dosage. In order to save labor costs, the frequency of pH detection is low (usually once an hour). While, it only needs 15 to 20 min for the pH regulator to flow from the adding point to the detecting point. So, low detection frequency results in control delay. Moreover, there might be fluctuation for the amount of dosage, ore input, water addition and the pH value of the circulating water, which increases the uncertainty of the pH control and the difficulty to stabilize the pH value within a given range. So, how to predict accurately and give a real-time adjustment is of great significance to stabilize the pH value and optimize the dosage.

3. Prediction Model of Slurry pH Based on Mechanism and Error Compensation

Bauxite reacts with alkali under high temperature, which is right the mechanism of bauxite digestion process. And, according to [20, 21], bauxite still reacts with alkali in solution under low temperature, and it is worth noting that the reaction rate is much slower. Meanwhile,

according to the experience of the operators, different ores need different amounts of pH regulator to get the same pH value. So, in this study, it is assumed that the pH regulator added into the slurry can be divided into two parts, that is, part of it reacts with some minerals, and the rest hydrolyzes and makes the slurry alkaline. Here, the second-order hydrolysis of the alkali (Na_2CO_3) can be ignored since it is much weaker than the first-order hydrolysis. Synthesizing the first-order hydrolysis of alkali and the mechanism of reaction between minerals and alkali, a mechanism-based prediction model for pH value is derived. While, there are large errors in the mechanistic model due to the complexity of the flotation mechanism and some uncertain factors, such as the fluctuation of temperature. An error compensation model using the ARMA method is then established to compensate the mechanistic model. According to the variation of the operating conditions, expert rules are presented to correct the error compensation direction. Revised error compensation value is then used to compensate the mechanism-based prediction model, and the final predicted pH value is obtained. The prediction model for slurry pH based on mechanism and error compensation is shown in Fig. 1.

3.1. Consumption of alkali by minerals based on experiment and industrial data

The reaction between minerals and alkali would consume part of alkali, that is to say alkali in the slurry cannot be fully used to adjust the pH value. The ore is composed of diasporic bauxite, chlorite, iron minerals, titanium minerals, illite, and so on. Ore compositions are complicated, so does the reaction of ore with alkali. Thus, two different experiments are designed as close as possible to the industrial grinding and classification process to obtain the consumption of alkali in the reaction between ore and alkali.

The ore samples used for the two experiments were collected from the feeding belt of the grinding mill. 11 ore samples were collected in 11 successive days with one sample a day. They all have different compositions, and therefore different properties.

3.1.1. Experiment 1

For the first experiment, the ore samples were crushed and sampled (with the same mass of 400 g) and then ground to the fineness with –200 mesh of 75% to 85%. Meanwhile, alkali was added and ground at the same time with the ore. After 20 min of grinding, the pH value of the slurry was tested, and the alkali consumed by the ore according to first-order hydrolysis was calculated, as the results shown in Figs. 2 and 3, respectively.

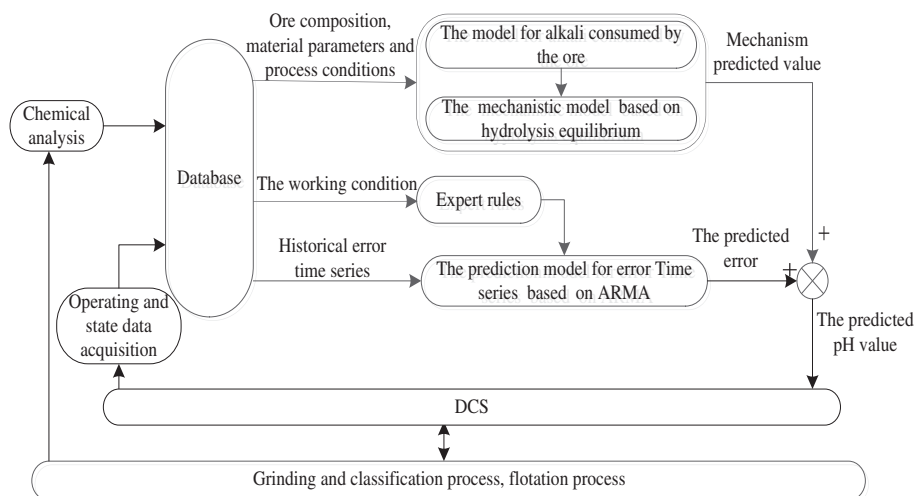


Fig. 1. The integrated prediction model for slurry pH.

From Figs. 2 and 3, one can see that pH values of different ores after grinding are different. For the same kind of ore, more alkali was added and more of it will be consumed by the ore, and that means the dissolution of ore is related to the pH value and it influences the pH value in verse. It also can be seen that, the trend for consumption of alkali is the same for different ores with different amounts of alkali added, that means the alkali consumption is inherently related to the ore properties.

3.1.2. Experiment 2

The process of the second experiment is as follows. The ore was crushed and then ground to the same fineness as the slurry for the actual flotation process is, that is the mass fraction of the -200 mesh particles is more than 85%. Then, three samples with 500 g of the ore were prepared and water was added to obtain the slurry for test with a concentration of 45%. And, the slurry temperature was kept at 38°C which is close to the temperature of the industrial slurry. Next, different amounts of alkali ($2200\text{ g}\cdot(\text{t ore})^{-1}$, $2500\text{ g}\cdot(\text{t ore})^{-1}$, $2800\text{ g}\cdot(\text{t ore})^{-1}$) were added into the three slurry samples, respectively. pH value was then detected every 5 min in 1 h.

When different amounts of alkali were added to the same kind of ore slurry with a concentration of 45%, the pH values varies with time, as shown in Fig. 4. It can be seen that the pH value decreases with time, while the decreasing rate becomes more and more slow.

3.1.3. Model regression

Finally, the 11 ore samples with different compositions were tested by the two experiments. The pH of each kind of ore with different amounts of alkali added is denoted as $P_{i,m,n}$ ($i = 1, 2, \dots, 11$; $m = 1, 2, 3$; $n = 1, 2, \dots, 12$). With the pH value, the amount of hydrolyzable alkali (denoted as $O_2\text{ g}\cdot(\text{t ore})^{-1}$) is calculated as follows,

$$O_2 = \frac{106V_{\text{water}} \left[\left(2 \times 10^{(-14+P_{i,m,n})} + K_1 \right)^2 - K_1^2 \right]}{5 \times 10^{-4} \cdot 4K_1} \quad (1)$$

where $K_1 = 1.8 \times 10^{-4}$ is the first-order hydrolysis equilibrium constant of Na_2CO_3 , $106\text{ (g}\cdot\text{mol}^{-1})$ is the molar mass of Na_2CO_3 , and V_{water} is the amount of water addition, whose value is 0.61 L according to the pulp density.

Denote $O_1\text{ (g}\cdot(\text{t ore})^{-1})$ as the initial alkali addition, the consumption of alkali by the reaction between it and the ore is as follows,

$$O_h = O_1 - O_2$$

$$O_h\text{ (mol}\cdot(\text{t ore})^{-1}) = O_h\text{ (g}\cdot(\text{t ore})^{-1})/106\text{ (g}\cdot\text{mol}^{-1}) \quad (2)$$

Because, there is a 15 to 20 min time delay in the actual industrial process, the amount of hydrolyzable alkali (denoted as $O_2\text{ g}\cdot(\text{t ore})^{-1}$) at 20 min of reaction is calculated for regression modeling. Finally, 33 sets of alkali consumption data were obtained, where, for

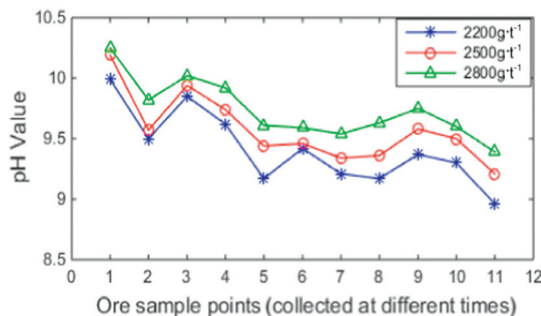


Fig. 2. pH value of slurries of different ores with different amounts of alkali.

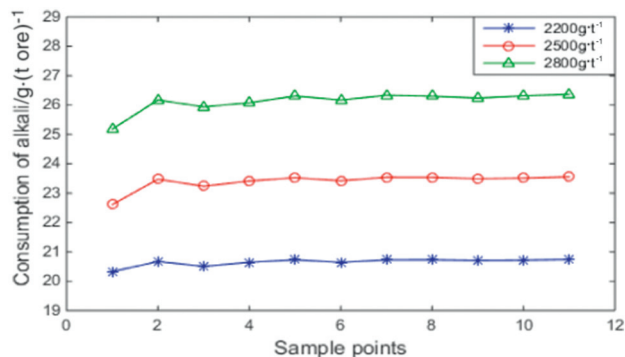


Fig. 3. The amount of alkali consumed by the ores in Fig. 2.

each kind of ore sample, there are 3 data sets with different amounts of alkali addition.

Meanwhile, the data from actual production process which corresponds to the ores in the two experiments were collected. The data of recycling water addition, new water addition, alkali addition and ore feed flowrate were recorded once an hour. While, the compositions of the ore were only analyzed every 8 h. So, the operating data of the first 2 h which are closest to the time when the ore composition is analyzed are averaged for the following model regression. For example, the ore is analyzed at 9:00 o'clock, then the operating data recorded at 9:00 and 10:00 are averaged. Therefore, 3 data sets were obtained each day, and totally 33 data sets were prepared for modeling. Then, the regression model for alkali consumption can be obtained based on experiment and industrial data-fitting.

According to the chemical composition of the flotation slurry and that of the ore before grinding, it is discovered that some of the main chemical compositions of the bauxite changes in the grinding and classification process. Among them, Al_2O_3 , SiO_2 and water would react with Na_2CO_3 to form sodium alumina-silicate ($\text{Na}_2\text{O}\cdot\text{Al}_2\text{O}_3\cdot 1.7\text{SiO}_2\cdot n\text{H}_2\text{O}$), part of the K_2O and Na_2O would dissolve in water, while they can be ignored for their contents are very small. Fe_2O_3 , TiO_2 and MgO are insoluble in water and they do not react with alkali. While CaO is dissolved in water to form Ca(OH)_2 precipitation. At the same time, the more the alkali is added in, the more the alkali is consumed by the ore. Therefore, without considering other chemical reactions, the regression model of alkali consumption is then obtained as follows from the experiment,

$$O = -9.823 + 0.365X_1 - 0.13X_2 + 0.574C \quad (R^2 = 0.992) \quad (3)$$

where $O\text{ (mol}\cdot(\text{t ore})^{-1})$ is consumption of Na_2CO_3 by 1 ton of ore, X_1 (%) is percentage of Al_2O_3 in the ore, X_2 (%) is percentage of SiO_2 in the ore, and $C\text{ (mol}\cdot(\text{t ore})^{-1})$ is the amount of Na_2CO_3 addition in 1 ton of ore.

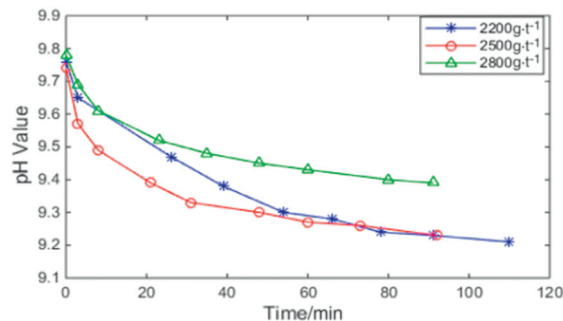


Fig. 4. pH value of the bauxite slurry with different amounts of alkali.

Assume that the consumption of alkali by the reaction between the alkali and the ore in the actual production process is the same with the consumption in the experiments, because the experiments were designed as close as possible to the actual production. Then, using Eq. (3), we can calculate the alkali consumption in the actual production using the data collected in the industrial process. However, one cannot ignore that in the actual process, addition of the recycling water (usually its pH > 8.3) means that more Na_2CO_3 is added in the slurry. So, according to its pH value, the recycling water is converted to equivalent amount of alkali when doing the calculation. The regression model is then modified and obtained as follows,

$$O = 25.917 + 0.0947X_1 - 0.107X_2 + 0.36C - 1.91p_{\text{Hw}} \quad (R^2 = 0.867) \quad (4)$$

where, p_{Hw} is the pH value of the circulating water, the meaning of the other symbols is the same with they are in Eq. (3).

It is difficult to build an accurate regression model for this kind of process and also difficult to validate it. On the one hand, because the size of the data set is limited. On the other hand, the composition of the ore is complex and the mechanism is not clear, so that the actual consumption of alkali by the ore in the industrial process cannot be determined. However, according to the mechanism in the digestion process, the reaction between Al_2O_3 , SiO_2 and the alkali must happen and consume some alkali. So, 8 samples of ore slurry before they go into the flotation process were collected and the content of sodium alumina-silicate (mass percentage of the solids) in the slurry was analyzed. It is assumed that Na in the sodium alumina-silicate is all from the alkali, the content of sodium alumina-silicate can be converted to the amount of alkali consumed by the reaction between the Al_2O_3 , SiO_2 and the alkali. The consumed alkali calculated by the regression model and the sodium alumina-silicate are plotted in Fig. 5. It can be seen that the results of the two calculation methods have the same trend, which shows that the regression model is reasonable.

3.2. Mechanism-based prediction model for pH value based on hydrolysis equilibrium

Except for the part of alkali that reacts with some compositions of the bauxite, the rest of alkali is considered to hydrolyze in water. Meanwhile, the addition of circulating water makes the slurry alkaline, which has an effect on the hydrolysis of alkali. Through the above analysis, a mechanism-based prediction model of pH value is investigated based on the principle of the first-order hydrolysis equilibrium. In the following, the modeling based on the hydrolysis process is introduced.

For the flotation process, the mass concentration of a pH regulator (Na_2CO_3) is denoted by W ($\text{g}\cdot\text{L}^{-1}$) and the addition rate is L ($\text{L}\cdot\text{min}^{-1}$), the average feed rate of ore in a short time (about 20 min) is F ($\text{t}\cdot\text{h}^{-1}$), the amount of circulating water added between the pH

regulator addition point and the pH detection point is Q_1 ($\text{t}\cdot\text{h}^{-1}$), the pH value of circulating water is represented by p_{Hw} , and the pure water is added in a rate of Q_2 ($\text{t}\cdot\text{h}^{-1}$), the concentration of OH^- in circulating water is denoted by C_2 and it can be calculated as follows,

$$C_2 = 10^{(-14+p_{\text{Hw}})} \quad (5)$$

Assume that all of the rest of the alkali will hydrolyze in water, and denote the OH^- concentration by X ($\text{mol}\cdot\text{L}^{-1}$) at the pH detection point. It can be obtained by the hydrolysis equilibrium,

$$\frac{(X + C_2)X}{C_1 - X} = K_1 \quad (6)$$

And then,

$$X = \frac{1}{2} \left[-(K_1 + C_2) + \sqrt{(K_1 + C_2)^2 + 4K_1C_1} \right] \quad (7)$$

where $K_1 = 1.8 \times 10^{-4}$ is the first-order hydrolysis equilibrium constant of Na_2CO_3 , C_1 ($\text{mol}\cdot\text{L}^{-1}$) is the concentration of the remaining alkali dissolved in all added water, which is calculated as,

$$C_1 = \frac{60WL - 106OF}{106[(Q_1 + Q_2) \times 10^3 + 60L]} \quad (8)$$

where 106 ($\text{g}\cdot\text{mol}^{-1}$) is the molar mass of Na_2CO_3 , 60 is the value which converts $\text{g}\cdot\text{min}^{-1}$ into $\text{g}\cdot\text{h}^{-1}$.

According to the total concentration of OH^- in solution, the pH value of the slurry using mechanism-based prediction can be obtained as,

$$p = 14 + \lg(X + C_2) \quad (9)$$

3.3. Error time series compensation based on ARMA

There are many factors which affect the flotation process and assumptions in the mechanism analysis process. Therefore, it is difficult to meet the production requirements by only using the mechanism model, thus the error compensation model is proposed. The time series of the error, which are the differences of the measured pH values and the predicted pH values of the mechanistic model, have certain relevance in time. Therefore, the ARMA method is used for the error prediction here. Time series of the measured pH value and the predicted pH value by mechanistic model are denoted by r_t ($t = 1, 2, \dots, n$) and p_t ($t = 1, 2, \dots, n$), respectively. Then, the time series of error is obtained by the subtraction, $e_t = r_t - p_t$, which is denoted as $\{e_1, e_2, \dots, e_n\}$.

The ARMA model is then established in three steps. The first step is to recognize the stationary of the model, the second is to determine the model order and the third is model parameter estimation.

In the ARMA modeling method, the error at time t is predicted using Eq. (10) by using time series $\{e_1, e_2, \dots, e_t, \dots, e_n\}$,

$$\hat{e}_t = \varphi_1 e_{t-1} + \varphi_2 e_{t-2} + \dots + \varphi_p e_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (10)$$

and the residual of the model is,

$$\delta = e_t - (\varphi_1 e_{t-1} + \varphi_2 e_{t-2} + \dots + \varphi_p e_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q}) \quad (11)$$

where e_t is the sample value, $\hat{e}_t$ is the output of the ARMA model, φ and θ represent autoregressive parameters and moving average parameters, respectively; p and q represent autoregressive model and moving average model order, respectively, and ε_t is white noise time series.

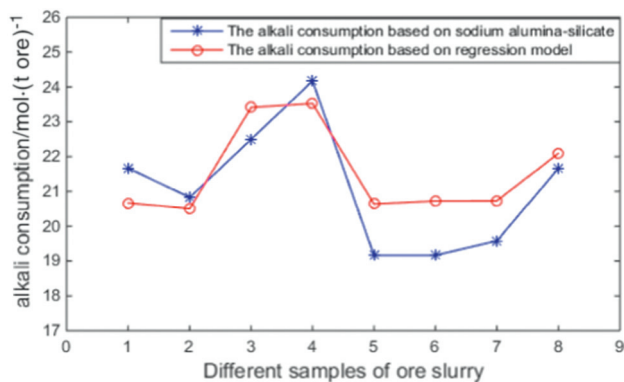


Fig. 5. The consumed alkali calculated by the regression model and the sodium alumina-silicate.

The time series of error $\{e_1, e_2, \dots, e_t, \dots, e_n\}$ are used for model training. Firstly, the ADF (augmented Dickey–Fuller) unit root test method is applied to recognize the stationary of the error sequence. There are three types of inspection formula for test,

$$\Delta e_t = \mu e_{t-1} + \sum_{j=1}^p \gamma_j \Delta e_{t-j} + \varepsilon_t \quad (12)$$

$$\Delta e_t = \alpha_0 + \mu e_{t-1} + \sum_{j=1}^p \gamma_j \Delta e_{t-j} + \varepsilon_t \quad (13)$$

$$\Delta e_t = \alpha_0 + \alpha_1 t + \mu e_{t-1} + \sum_{j=1}^p \gamma_j \Delta e_{t-j} + \varepsilon_t \quad (14)$$

where, ∂_0 is a constant, t denotes time, ∂_1 is the coefficient of t , μ is the coefficient of e_{t-1} , p is the lag order of Δe_t , γ_j is the coefficient of Δe_t with different lag order, and ε_t is white noise.

The null hypothesis and the alternative hypothesis are,

H₀. $\mu = 0$ (e_t is non-stationary).

H₁. $\mu < 0$ (e_t is stationary).

In **H₀** hypothesis, it is assumed that the error series e_t has a unit root and it is non-stationary. That is to say, if **H₀** hypothesis is accepted, e_t is non-stationary; otherwise, it is stationary. In **H₁** hypothesis, it is assumed that e_t is stationary.

The ADF test is used to estimate the three Eqs. (12) to (14) at the same time by using the least square method, and the T statistics of μ is then calculated, which is marked as τ . A threshold can be obtained by looking-up table according to the significance level. If $\tau >$ threshold, then **H₀** is accepted, that is to say e_t has a unit root and it is non-stationary. If $\tau <$ threshold, then **H₀** is unaccepted, that is to say there does not exist a unit root and e_t is stationary.

Only when all the three models accept **H₀**, the time series are considered to be a non-stationary process. As long as one of the inspection formulas rejects **H₀**, the time series is a stationary process. It is discovered that all three inspection formulas of the error time series used for modeling reject **H₀**, indicating that there is no unit root. So, the error time series is a stationary process.

Then, autocorrelation function (ACF) and partial autocorrelation function (PACF) methods are applied to determine the model order. The trailing and truncation of ACF and PACF are investigated. When the ACF and PACF for time series are of p -order trail and q -order trail, respectively, the series is modeled as ARMA (p, q). Here, by calculating the autocorrelation of the error series, the ACF of error time series is 6-order trail and the PACF is 1-order trail, thus the model of the error time series is ARMA (6,1).

The least square method is used for parameter identification.

Let,

$$\begin{aligned} \beta &= [\varphi_1 \quad \dots \quad \varphi_p \quad 1 \quad \theta_1 \quad \dots \quad \theta_q]^T \\ H &= \begin{bmatrix} e_{t-1}^{(1)} & \dots & e_{t-p}^{(1)} & \varepsilon_t^{(1)} & -\varepsilon_{t-1}^{(1)} & \dots & -\varepsilon_1^{(1)} \\ \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ e_{t-1}^{(m)} & \dots & e_{t-p}^{(m)} & \varepsilon_t^{(m)} & -\varepsilon_{t-1}^{(m)} & \dots & -\varepsilon_{t-q}^{(m)} \end{bmatrix} \\ Z &= [e_t^{(1)} \quad e_t^{(2)} \quad \dots \quad e_t^{(m)}]^T \\ Y &= [\delta^{(1)} \quad \delta^{(2)} \quad \dots \quad \delta^{(m)}]^T \end{aligned} \quad (15)$$

where, m is the number of samples.

Criterion function is defined as,

$$J = \frac{1}{2} Y^T Y \quad (16)$$

Estimation solution of the least square method is,

$$\beta = (H^T H)^{-1} H^T Z \quad (17)$$

From the training data, $\beta = [0.7954; -0.1246; 0.3018; 0.0878; -0.1; -0.1128; 1; -0.2841]$ was obtained. Put β into formula (10), the ARMA model is as follows,

$$\hat{e}_t = 0.7594e_{t-1} - 0.1246e_{t-2} + 0.3018e_{t-3} + 0.0878e_{t-4} - 0.1e_{t-5} - 0.1128e_{t-6} + \varepsilon_t - 0.2841\varepsilon_{t-1} \quad (18)$$

3.4. Error compensation direction correction based on expert rules

Mechanism-based model reflects the influence of the OH^- concentration on pH value, due to the hydrolysis of alkali and the addition of circulating water in flotation process, while the premise is that the chemical environment is stable. In the mechanistic modeling process, it is assumed that all of the rest of the alkali hydrolyzes in the water under first-order. However, the hydrolysis of alkali is complex. It is influenced by the amount of alkali added, the feed rate of the ore and other factors. Therefore, the pH value of the slurry is affected by process variables, such as the addition of pure water and circulating water, the OH^- concentration in circulating water, the variation of temperature, and the mineralogy characteristics of ore. When ore properties are relatively stable, fluctuation of process conditions is the main reason for the change of pH value. Meanwhile, current predicted value from the ARMA model is related to previous moments, which cannot reflect the sudden fluctuations of process condition. That is to say, error time series may compensate in the opposite direction. However, experienced workers can judge the pH change directions accurately when the working conditions change obviously. Thus expert rules are established to judge error compensation direction based on variations of process conditions and experiential knowledge. When compensation direction of the expert rule is consistent with $\hat{e}_t$, $\hat{e}_t$ is used to compensate mechanism model directly, otherwise, compensation direction is subject to the output of the expert rules with absolute value equaling to $\hat{e}_t$.

Experience has shown that a series of situations, such as the incremental of Na_2CO_3 addition, the incremental of ore input, the incremental of water addition and the incremental of pH value in circulating water, have great influence on the slurry pH. So, they are used as input attributes of the expert rules. Incremental is the difference of the current value and the previous value of a variable. The definition of the variables in the expert rules is listed in Table 1. The expert rules are as follows,

IF $\Delta\beta_t \geq 3.4$, THEN $k_t = 1$;
 IF $\Delta\beta_t \leq -4.7$, THEN $k_t = 0$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $\Delta p_t \leq -0.21$, THEN $k_t = 1$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $\Delta p_t \geq 0.23$, THEN $k_t = 0$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $-0.21 < \Delta p_t < 0.23$ AND $\Delta m_t \leq -100$, THEN $k_t = 1$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $-0.21 < \Delta p_t < 0.23$ AND $\Delta m_t \geq 100$, THEN $k_t = 0$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $-0.21 < \Delta p_t < 0.23$ AND $-100 < \Delta m_t < 100$ AND $\Delta\omega_t \geq 12$, THEN $k_t = 1$;
 IF $-4.7 < \Delta\beta_t < 3.4$ AND $-0.21 < \Delta p_t < 0.23$ AND $-100 < \Delta m_t < 100$ AND $\Delta\omega_t \leq -6$, THEN $k_t = 0$;
 ELSE k_t is in the same direction with $\hat{e}_t$.

Finally, predicted pH value after compensation can be expressed as,

$$\hat{p}_{t+1} = \begin{cases} p_{t+1} + \hat{e}_{t+1} & k_{t+1} \text{ in the same direction with } \hat{e}_{t+1} \\ p_{t+1} + (-1)^{k_{t+1}+1} |\hat{e}_{t+1}| & k_{t+1} \text{ in the different direction with } \hat{e}_{t+1} \end{cases} \quad (19)$$

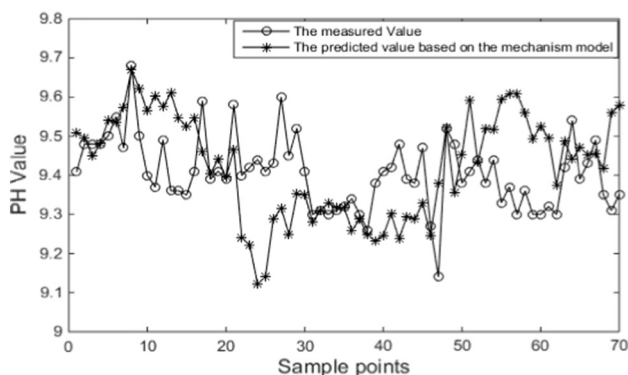
Table 1

Definition of the variables in the expert rules

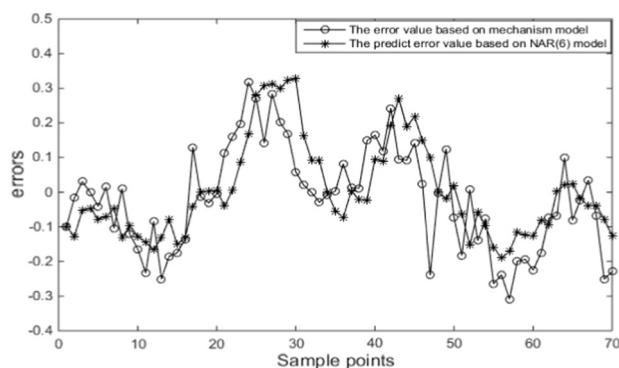
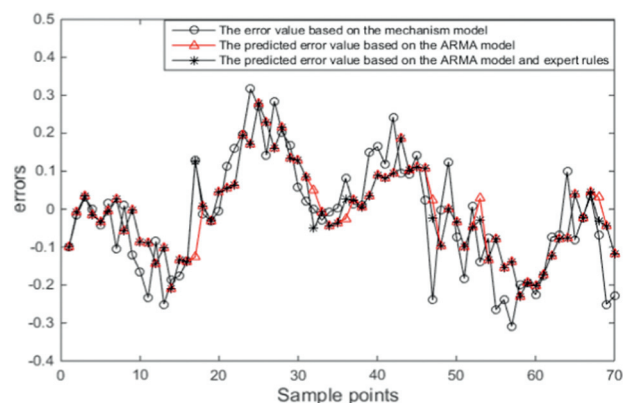
Variables	Definition
$\Delta\beta_t/t$	The incremental of ore input at time t .
Δp_t	The incremental of pH value in circulating water at time t .
$\Delta m_t/g \cdot t^{-1}$	The incremental of Na_2CO_3 addition at time t .
$\Delta\omega_t/t$	The incremental of water addition at time t .
k_t	The output of the expert model, and value 1 and 0 represent increasing and decreasing of the pH value, respectively.
p_{t+1}	The pH value based on mechanism model at time $t+1$.
$\hat{e}_{t+1}$	The predicted value of ARMA model at time $t+1$ for error compensation.
k_{t+1}	The output of the expert model at time $t+1$.
$\hat{p}_{t+1}$	The output pH value based on the integrated model at time $t+1$.

4. Simulation Results Using Industrial Data

The samples were collected at an alumina production plant with the sampling interval of half an hour. These data are different from the data used in Section 3. The first 215 samples are used for training, and the following 70 groups of data in three days' continuous production are used for model validation. A Nonlinear Autoregressive (NAR) neural network model is compared with the ARMA model, and the results are shown in Figs. 6 to 10.

**Fig. 6.** The pH results of the mechanistic model.

From Fig. 6, the results show that the mechanism model has direct response to process variables. Based on the variation of process variables, it can reflect the trend of the measured value in a way. However, the errors are large and it cannot meet the actual needs of the industrial production. Therefore, the error prediction model is established to compensate the mechanistic model, and the results are shown in Figs. 7 and 8. Here, a NAR neural network with 6 inputs and 10 hidden layer nodes is compared, and the maximal absolute error is 0.3298. The maximal absolute error of the ARMA model is 0.2624. We can learn that the ARMA

**Fig. 7.** The prediction error of the NAR (6) model.**Fig. 8.** The prediction error of the ARMA model and expert rules.

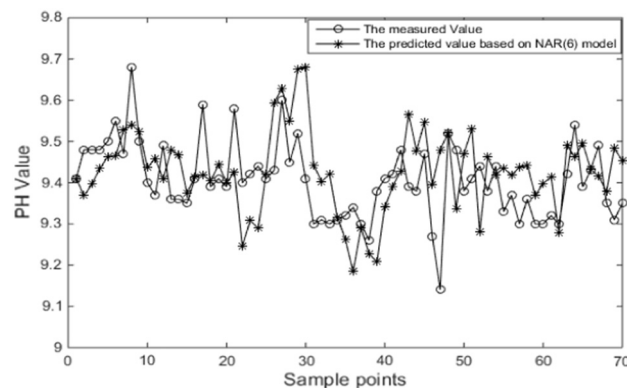
method has a better performance than the NAR neural network on this question. That is to say, time series of the error is more likely linear. Meanwhile, the former is simpler than the latter. In Fig. 8, we can see that expert rules based on operators' experience avoid the situation when the error compensation direction of the ARMA model is wrong because of the large changes of process variables. It corrects the compensation direction of 6 points, which reduces the error of these points to some extent. The final predicted values of slurry pH are shown in Figs. 9 and 10, in which we can find that the integrated model has higher prediction accuracy. The comparison of the performance of different models for slurry pH prediction is shown in Table 2, where the mean absolute error (MAE) and root mean square error (RMSE) are calculated in Eqs. (20) and (21), where y and $\hat{y}$ are the real and the predicted value, respectively; n is the number of the samples.

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (20)$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (21)$$

5. Conclusions

This paper presents an integrated prediction model for slurry pH in bauxite froth flotation. Because the alkali consumption is related to the compositions of the ore, a regression model for alkali (Na_2CO_3) consumed by the ore is proposed based on the reaction between ore and alkali. Considering the hydrolysis of the remaining alkali which makes the slurry alkaline, a mechanism-based prediction

**Fig. 9.** The pH results of the NAR (6) model.

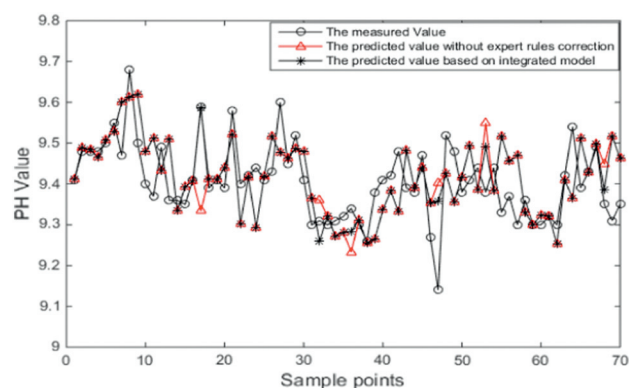


Fig. 10. The pH results of the integrated model.

Table 2

Performance comparison of different models for slurry pH prediction

Models	MAE	RMSE
Mechanism model	0.1190	0.1488
Mechanism and NAR model	0.0891	0.1094
Mechanism and ARMA model	0.0704	0.0984
The integrated model	0.0637	0.0935

model is established for the pH value. As the time series of the error have certain relevance in time, an ARMA model is built to predict the error which uses the errors of the mechanism model as inputs. When the production conditions fluctuate suddenly, the ARMA model cannot predict the pH change directions accurately but experienced workers can. Thus, expert rules are proposed to correct the error compensation direction. Finally the integrated prediction model is verified by the industrial data. The validation results show that the presented model meets the requirements of industrial production. It is worth noting that the model for alkali consumed by the ore is a regression model which considered different kinds of ores, therefore, it has a greater adaptability in the industrial process.

In conclusion, the integrated model can achieve remarkable performance for slurry pH prediction, which laid the foundation for the predictive control of a pH regulator.

References

- [1] X. Ma, W.J. Bruckard, R. Holmes, Effect of collector, pH and ionic strength on the cationic flotation of kaolinite, *Int. J. Miner. Process.* 93 (1) (2009) 54–58.
- [2] S.W. Huang, B.Y. Hang, J. Hu, Y.Y. Deng, Z.Q. Zheng, The application of pH apparatus in flotation, *Nonferrous Metals Miner. Process. Sect. 2* (2008) 41–45.
- [3] Q.W. Qin, Y.H. Hu, Dissolution of sparingly soluble minerals and its effect on floatability of minerals, *Min. Metall. Eng.* 2 (1999) 30–33.
- [4] A.M. Vieira, A.E.C. Peres, The effect of amine type, pH, and size range in the flotation of quartz, *Miner. Eng.* 20 (10) (2007) 1008–1013.
- [5] Z.Y. Li, H.M. Gu, Y.C. Zhai, Effect of pH value on coprecipitation recovery rates of W–Cu (70 g:30 g), *Rare Metal Mater. Eng.* 37 (3) (2008) 460–463.
- [6] W.P. Liu, X. Hui, X.Y. Yang, X.C. Shi, Extraction of molybdenum from low-grade Ni–Mo ore in sodium hypochlorite solution under mechanical activation, *Miner. Eng.* 24 (14) (2011) 1580–1585.
- [7] P. Kadlec, B. Gabrys, Adaptive on-line prediction soft sensing without historical data, *International Joint Conference on Neural Networks IEEE Xplore* 2010, pp. 1–8.
- [8] H. Kaneko, K. Funatsu, Adaptive soft sensor model using online support vector regression with the time variable and discussion on appropriate hyperparameters and window size, *Comput. Chem. Eng.* 58 (45) (2013) 288–297.
- [9] C.X. Liu, J.L. Ding, A.J. Toprac, T.Y. Chai, Data-based adaptive online prediction model for plant-wide production indices, *Knowl. Inf. Syst.* 41 (2) (2014) 401–421.
- [10] W.M. Shao, X.M. Tian, Soft sensor for nonlinear processes based on ensemble partial least squares with adaptive localization, *Control and Decision Conference, IEEE Xplore* 2015, pp. 737–742.
- [11] N. Sammaknejad, B. Huang, R.S. Sanders, L.A. Espejo, Adaptive soft sensing and on-line estimation of the critical minimum velocity with application to an oil sand primary separation vessel, *IfacPapersonline* 48 (8) (2015) 211–216.
- [12] R. Mallipeddi, P.N. Suganthan, Improved adaptive differential evolution algorithm with external archive, *Swarm, Evolutionary, and Memetic Computing, Springer International Publishing* 2013, pp. 170–178.
- [13] J. Deng, L. Xie, L. Chen, S. Khatibisepehr, B. Huang, F.W. Xu, *et al.*, Development and industrial application of soft sensors with on-line Bayesian model updating strategy, *J. Process Control* 23 (3) (2013) 317–325.
- [14] J.L. Ding, T.Y. Chai, H. Wang, Dynamic Prediction Model for Mixed Concentrate Grade of Mineral Processing Plant, *IEEE Conference on Decision and Control*, Vol. 58(8), 2010, pp. 6773–6778.
- [15] Mika Luukkainen, Eero Hälikkää, Teri Hiltunen, Yrjö Hiltunen, Dynamic soft sensors for NOx emissions in a circulating fluidized bed boiler, *ICAIE 2011 – International Conference on Applied Energy* 2011, pp. 483–490.
- [16] H.J. Galicia, Q.P. He, J. Wang, Comparison of the performance of a reduced-order dynamic PLS soft sensor with different updating schemes for digester control, *Control. Eng. Pract.* 20 (8) (2012) 747–760.
- [17] E. Putz, A. Cipriano, Hybrid Dynamic Predictive Model for Rougher Flotation, *Ifac Proceedings Volumes*, Vol. 46(16), 2013, pp. 155–160.
- [18] P.F. Cao, X. Luo, Modeling of soft sensor for chemical process, *CIESC J.* 64 (3) (2012) 788–800.
- [19] Z.J. Zeng, X.L. Wang, C.H. Yang, J. Wu, Multi-model based on image feature reduction for pH value soft-sensing in froth flotation process, *IWACIII 2013 - 3rd International Workshop on Advanced Computational Intelligence and Intelligent Informatics*, 2014.
- [20] S.W. Bi, *Alumina Production Process*, Chemical Industry Press Beijing, 2006.
- [21] W.T. Hu, H.J. Wang, C.Y. Sun, Y. He, C.L. Ji, G.L. Wang, *et al.*, Reaction mechanism between ferric bauxite and soda lime in reduction atmosphere, *J. Harbin Eng. Univ.* 34 (5) (2013) 662–668.